Lecture 10

Public Key Cryptography: Encryption + Signatures

Identification

- Public key cryptography can be also used for IDENTIFICATION
- Identification is an interactive protocol whereby one party: "prover" (who claims to be, say, Alice) convinces the other party: "verifier" (Bob) that she is indeed Alice
- Identification can be accomplished with public key digital signatures
- However, signatures reveal information…
- Also, signatures are "transferable", i.e., anyone can verify them

Fiat-Shamir Identification Scheme

- In Fiat-Shamir, prover has an RSA modulus n = pq (factorization is secret).
- Factors themselves are not used in the protocol.
- Unlike RSA, a trusted center can generate a global n, used by everyone, as long as nobody knows its factorization. Trusted center can "forget" the factorization after computing n.

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Fiat-Shamir Identification Scheme

- Secret Key: Prover (P) chooses a random value
  $1 < S < n$ (to serve as the key)
  such that gcd(S,n) = 1
- Public Key: P computes $I=S^2 \bmod n$, publishes (I,n) as his public key.
- Purpose of the protocol: P has to convince verifier (V) that he knows the secret S corresponding to the public key (I,n),
  - i.e., to prove that he knows a square root of I mod n, without revealing S or any portion thereof

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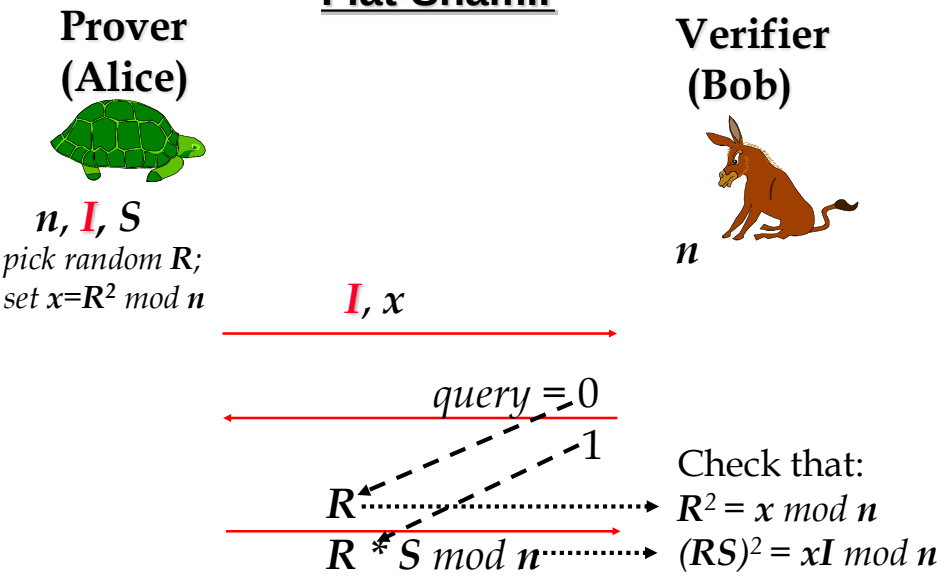
Fiat-Shamir



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Fiat-Shamir Identification Scheme

V wants to authenticate identity of P, who claims to have a public key I. Thus, V asks P to convince him that P knows the secret key S corresponding to I .

1. P chooses at random $1 < R < n$ and computes: $X = R^2 \bmod n$
2. P sends X to V
3. V randomly requests from P one of two things (0 or 1):
   (a) R
   or
   (b) RS mod n
4. P sends requested information

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Fiat-Shamir ZK Identification Scheme

5. V checks the correct answer:
   a) $R^2 \stackrel{?}{=} X \pmod n$

      or

   b) $(R*S)^2 \stackrel{?}{=} X*I \pmod n$
6. If verification fails, V concludes that P does not know S
7. Protocol is repeated t (usually 20, 30, or log n) times, and, if each one succeeds, V concludes that P is the claimed party.

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What if Prover knows the challenge ahead of time: Case 0

n, ***I*** *(doesn't know S)*

pick random ***R****;*
set $x = R^2 \bmod n$

n

I, ***x*** →

← *query* = 0

R →

Check that:
$R^2 = x \bmod n$

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What if Prover knows the challenge ahead of time: Case 1

n, ***I*** *(doesn't know S)*
pick random ***R****;*
set $x=R^2*I \bmod n$

$I, x=R^2*I$ →

← *query* = 1

$R*I \bmod n$ →

Check that:
$(R*I)^2 = x*I \bmod n$

(Instead of: R*S mod n)

Fiat-Shamir Identification Scheme

CLAIM: Protocol does not reveal ANY information about S or

Protocol is **ZERO-KNOWLEDGE**

Proof: We show that no information on S is revealed:

- Clearly, when P sends X or R, he does not reveal any information on S.
- When P sends *RS mod n*:
  - RS mod n is random, since R is random and gcd(S, n) = 1.
  - If adversary can compute any information on S from

$$I, n, X \text{ and } RS \bmod n$$

he can also compute the same information on S from I and n, since he can choose a random $T = R'S \bmod n$ and compute:

$$X' = T^2I^{-1} = (R')^2S^2I^{-1} = (R')^2$$

Security

Clearly, if P knows S, then V is convinced of his identity.

If P does not know S, he can either:

1. know R, but not RS mod n. Since he is choosing R, he cannot multiply it by the unknown value S or
2. choose RS mod n, and thus can answer the second question: RS mod n. But, in this case, he cannot answer the first question R, since he needs to divide by the unknown S.

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Security

- In any case, adversary cannot answer both questions, since otherwise he can compute S as the ratio between the two answers.
- But, we assumed that computing S is hard, equivalent to factoring n.
- Since P does not know in advance (when choosing R or RS mod n) which question that V will ask, he cannot foresee the required choice. He can succeed in guessing V's question with probability 1/2 for each question.
- The probability that V fails to catch P in all runs is thus: 2^{-t} (e.g., 1 in 1,000,000,000 for t=20)

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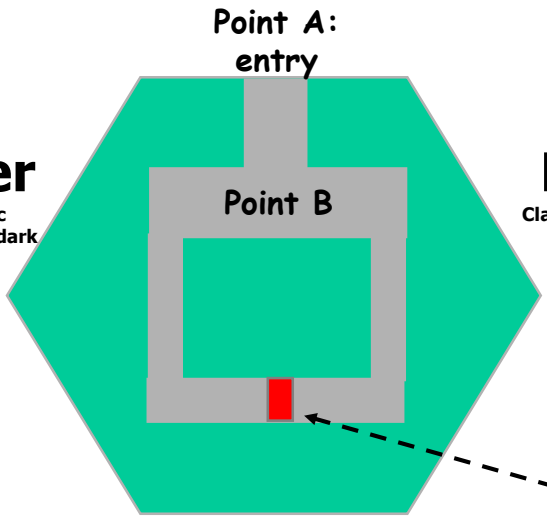
How to explain ZK to your children

Verifier

Claustrophobic and afraid of the dark



Prover

Claims to have the key

V cannot follow P into the cave

Locked door on both sides

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How to explain ZK to your children

The Protocol:

1) V asks someone he trusts to check that the door is locked on both sides.

2) P goes into the maze past point B (heading either right or left)

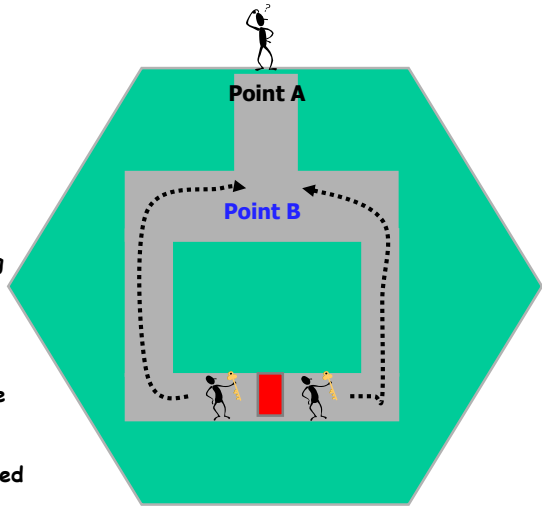
3) V looks into the cave (while standing at point A)

4) V randomly picks right or left

5) V shouts (very loudly!) for P to come out from the picked direction

6) If P doesn't come out from the picked direction, V knows that P is a liar and protocol terminates

REPEAT (2)-(6) n TIMES



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